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# Concept Modeling with Superwords

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## Abstract

In information retrieval, a fundamental goal is to transform a document into concepts that are representative of its content. The term “representative” is in itself challenging to define, and various tasks require different granularities of concepts. In this paper, we aim to model concepts that are sparse over the vocabulary, and that flexibly adapt their content based on other relevant *semantic* information such as textual structure or associated image features. We explore a Bayesian nonparametric model based on nested beta processes that allows for inferring an unknown number of strictly sparse concepts. The resulting model provides an inherently different representation of concepts than standard LDA (or HDP) based topic models, and allows for direct incorporation of semantic features. We demonstrate the utility of this representation on multilingual blog data and the Congressional Record.

## 1 Introduction

Information overload is a ubiquitous problem that affects nearly all members of society, from researchers sifting through millions of scientific articles to Web users trying to gauge public opinion by reading blogs. Even as information retrieval (IR) methods evolve to move beyond the traditional Web search paradigm to more varied retrieval tasks focused on combating this overload, they remain reliant on suitable document representations. While all representations ultimately distill the contents of a document collection into fundamental *concepts*, representing the atomic units of

information, a particular choice of representation can have potentially drastic consequences on performance.

For instance, a common desideratum of retrieval tasks is *diversity* in the result set [5]. Here, the document representation must be expressive enough such that it is recognizable when two documents are about the same idea. A fine-grained representation (e.g., individual words or named entities) may lead to many concepts with the same meaning. For example, “President Obama,” “Barack Obama,” “Mr. President” and “POTUS” are all distinct named entities that refer to the 44th President of the United States, but may end up as distinct concepts depending on the representation. On the other end of the spectrum, coarse-grained representations, such as topics from a topic model [3], may conflate together many ideas that are only vaguely related. This vagueness is particularly a problem for systems that attempt to *personalize* results to user’s individual tastes, and as such need to estimate a user’s level of interest in each concept (e.g., [1, 6]).

In this paper, we seek to *learn* concept representations at an appropriate level of granularity, representing each concept as a set of words that are functionally equivalent for the particular task at hand. We take a cue from the computer vision community [15] and refer to such concepts as *superwords*. We desire the following characteristics from our model:

1. The number of potential ideas that are to be modeled as concepts is unknown and unbounded, and thus our model should be able to handle this uncertainty.
2. Our model should specify a probabilistic interpretation for how much each document is about any given concept, allowing for seamless incorporation into IR systems.
3. We should be able to easily encode semantic information about the vocabulary into our model, based on the idea that two words occurring in the same concept should share a meaning in some underlying semantic space.
4. The same concept can be represented differently

in different documents (i.e., it can contain slightly different sets of words).

Our approach addresses these properties by relying on the machinery of Bayesian nonparametric methods. In particular, we use a *nested beta process* prior to provide strict sparsity in the set of concepts used in a document and the set of words associated with each concept, while at the same time allowing for uncertainty in the number of concepts [9–11, 16, 18]. Such a prior encourages sharing of concepts and word choices among documents, but provides flexibility for documents to differ. For instance, Democrats may say “healthcare reform” while Republicans may opt to say “Obamacare,” but both are referring to the same concept. Previous models assume that topics are the same for each document, and so to model such a phenomenon, they either create multiple topics to refer to the same idea, or else a single conflated topic with probability mass on words informed by both populations. We avoid both undesirable options by using a more expressive prior.

Moreover, as described above, there is inherent uncertainty in the granularity of the concepts. Specifically, does one choose more concepts, each with fewer words, or fewer concepts, each with more words? This is a question of *identifiability* in the nested beta process. Thus, to further inform our sought-after sparsity structure, we augment the prior with a *semantic feature matrix* in which each word in the vocabulary has an associated observed feature vector. This matrix has the additional benefit of allowing us to fuse multiple sources of information. For example, the feature vector may capture sentence co-occurrence of words in the vocabulary. Such information harnesses the structure of the text lost in the simple bag-of-words formulation. Alternatively (or additionally), this feature vector can include non-textual information, such as features of images of each word, or features learned from user feedback that can be useful for personalization. The semantic feature matrix is modeled as a weighted combination of latent *concept semantic features*, where the weighting is based on word assignments to each concept across the corpus, thereby encoding semantic similarity into concept membership.

The model we propose in this paper is fundamentally different from topic models and other generative models of text in that we represent each concept as a *sparse set of words*, rather than as topics that are distributions over an entire vocabulary. As such, our concepts can be more coherent and focused than traditional topics. For example, the top row of Figure 1 compares a concept learned via our model with a corresponding topic from a hierarchical Dirichlet process (HDP) [17], both learned on the same Congressional Record cor-

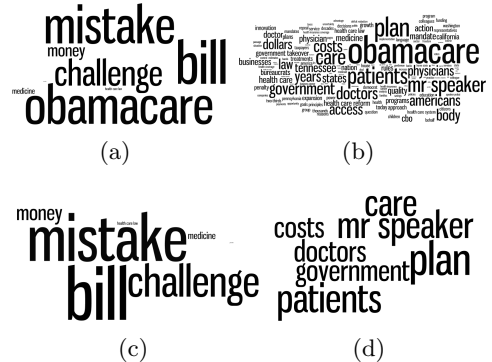


Figure 1: The top row contains word clouds comparing a concept learned using our model (a) with a topic from an HDP (b), both involving the word “Obamacare.” The bottom row has the same concept and topic, but as displayed to users in our user study, described in Sec. 5.

pus (cf. Sec. 5).<sup>1</sup> We see in (a) that our model is able to focus on the salient idea of the word “Obamacare,” in that it is a pejorative term used by Republicans to describe President Obama’s health care reform package. The HDP topic in (b) provides a much vaguer and more diffuse representation. Additionally, following Williamson et al. [21], our prior allows us to decouple the prevalence of a concept from its strength. In other words, in our model, it is possible for a rare concept to be highly important to the documents that contain it, which is a characteristic difficult to obtain in traditional topic modeling approaches.

To recap, the main contributions of this paper are:

- A novel use of a nested beta process prior to define concepts in a documents as sparse sets of words;
- The ability to elegantly incorporate semantic side information to guide concept formation;
- An MCMC inference procedure for learning concepts from data; and
- Empirical results—including a user study—on both multilingual blog data and the Congressional Record, showing the efficacy of our approach.

## 2 Nested Beta Processes

We wish to model the situation where there are an unknown (and unbounded) collection of concepts in the world, and each document is about some sparse subset of them. A natural way to model this uncertainty and unboundedness in a probabilistic manner is to look to Bayesian nonparametric methods. For instance, Dirichlet processes (DPs) have long been used as priors for mixture models in which the number of mixture components is unknown [2]. However, in our

<sup>1</sup>Throughout this paper, word clouds are used to illustrate topics and concepts, where the size of a word is proportional to its weight or prevalence in the topic or concept.

case, rather than assigning each observation to a single cluster as done in DPs, we require a *featural* model, where each document can be made up of several concepts. As such, we base our model on a nested version of the *beta process* [10], described in this section.

## 2.1 Beta Process - Bernoulli Process

We consider the situation where there is a countably infinite number of concepts in the world, each represented by a coin with a particular bias,  $\omega_j$ , and a set of attributes that define the concept,  $\psi_j$ . A process for assigning concepts to a particular document could thus be to flip each of the coins, and if coin  $j$  lands heads, we assign concept  $j$  to the document. This process of flipping coins to assign concepts to each document is known formally as the *Bernoulli process*, since we have a  $\hat{c}_j^{(d)} \sim \text{Bernoulli}(\omega_j)$  draw for each concept, where  $\hat{c}_j^{(d)}$  indicates that concept  $j$  is on for document  $d$ .

As we do not know the values of the coin biases  $\omega_j$  and concept attributes  $\psi_j$ , we wish to place a prior over them that encodes our desire for a sparse set of active concepts per document. Specifically, if we let,

$$B = \sum_{j=1}^{\infty} \omega_j \delta_{\psi_j}, \quad (1)$$

we exploit of the fact that the Bernoulli process has a conjugate prior known as the beta process, and write  $B \sim \text{BP}(b, B_0)$  to indicate that  $B$  is distributed according to a beta process with *concentration parameter*  $b > 0$  and a base measure  $B_0$  over some measurable space  $\Psi$ . By construction, the biases  $\omega_j$  lie in the interval  $(0,1)$ , and thus if the mass of the base measure,  $\alpha_\omega = B_0(\Psi)$ , is finite, then  $B$  has finite expected measure and we obtain our desired concept sparsity.<sup>2</sup>

Formally, the beta process is defined as a realization of a nonhomogenous Poisson process with rate measure defined as the product of the base measure  $B_0$  and an improper beta distribution. In the special case where the base measure contains discrete atoms  $i$ , with associated measure  $\lambda_i$ , then a sample  $B \sim \text{BP}(b, B_0)$  necessarily contains the atom, with associated weight  $\omega_i \sim \text{Beta}(b\lambda_i, b(1 - \lambda_i))$  (cf. [18]).

## 2.2 Indian Buffet Process

A Bernoulli process realization  $\hat{\mathbf{c}}^{(d)}$  from our prior determines the subset of concepts that are active for document  $d$ . As in Thibaux and Jordan [18], due to conjugacy, we can analytically marginalize the beta process

<sup>2</sup>It is important to note that, unlike a Dirichlet process base measure,  $B_0(\Psi)$  need not be equal to 1. The beta process is a *completely random measure* [12], where realizations on disjoint sets are independent random variables.

measure  $B$  and obtain a predictive distribution simply over the concept assignments  $\hat{\mathbf{c}}^d$ . Taking the concentration parameter to be  $b = 1$  yields the Indian buffet process (IBP) of Griffiths and Ghahramani [9].

The IBP is a culinary metaphor that describes how the sparsity structure is shared across different draws of the Bernoulli process. Each document is represented as a customer in an Indian buffet with infinitely many dishes, where each dish represents a concept. The first customer (document) samples a  $\text{Poisson}(\alpha_\omega)$  number of dishes. The  $d$ th customer selects a previously tasted dish  $j$  with probability  $m_j/d$ , where  $m_j$  is the number of customers to previously sample dish  $j$ . He then chooses a  $\text{Poisson}(\alpha_\omega/d)$  number of new dishes. With this metaphor, it is easy to see that the sparsity pattern over concepts is shared across documents because a document (customer) is more likely to pick a concept (dish) if many previous documents have selected it.

## 2.3 Nested Beta Process

Above, we described how a beta process prior is well-suited to modeling the presence or absence of concepts in each document  $d$  of a document collection. However, we are still left with the task of modeling the presence or absence of words in a particular concept. Just as was the case at the concept level, a document should be more likely to activate a word  $i$  in a concept  $j$  if many other documents also have word  $i$  active in concept  $j$ .

This leads to a natural extension of the IBP culinary metaphor to include condiments that are added alongside each dish. Specifically, after the  $d$ th customer (document) selects her dishes (concepts) from the Indian buffet, she selects an assortment of *chutneys* (words) to accompany each dish. Analogous to what happens at the concept level, if a customer is the first to sample from dish  $j$ , then she selects  $\text{Poisson}(\alpha_\gamma)$  types of chutney. The  $d$ th customer to sample from dish  $j$  selects chutney  $i$  with probability  $m_{ji}/d$ , where  $m_{ji}$  is the number of previous customers who sampled chutney  $i$  alongside dish  $j$ . She then selects  $\text{Poisson}(\alpha_\gamma/d)$  types of new chutney.

Formally, this results in a *nested* beta process (nBP),

$$B \sim \text{BP}(b_1, \text{BP}(b_2, B_0)), \quad (2)$$

which can equivalently be described as ,

$$B = \sum_{j=1}^{\infty} \omega_j \delta_{B_j^*}, \quad B_j^* = \sum_{i=1}^{\infty} \gamma_{ji} \delta_{\theta_{ji}}. \quad (3)$$

That is, a draw from a nBP is a discrete measure whose atoms are themselves discrete measures. Just as Bernoulli process draws from the top-level beta

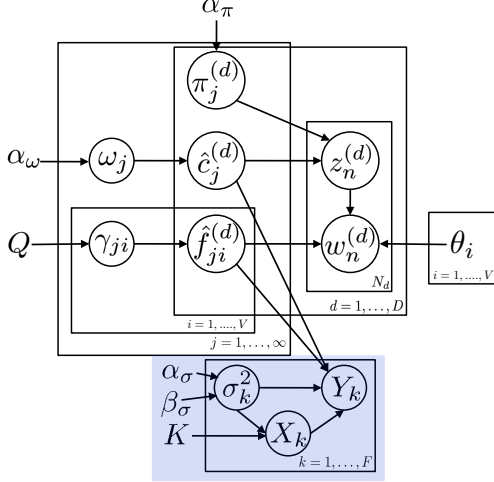


Figure 2: Plate diagram and generative model. We write  $(\omega, \gamma) \sim nBP(\alpha_\omega, Q)$  for the distribution over the coin biases.

process results in concept assignments  $\hat{c}_j^{(d)}$ , word assignments for concept  $j$  are obtained by sampling  $\hat{f}_{ji}^{(d)} \sim \text{Bernoulli}(\gamma_{ji})$  from the lower level beta process  $B_j^*$ , where  $\hat{f}_{ji}^{(d)}$  indicates whether word  $i$  is active in concept  $j$  for document  $d$ . A related idea of nesting beta processes is discussed in [11].

It is important to note that, in analogy to hierarchical clustering (e.g., the nested Dirichlet process of [16]), the nBP is only weakly identifiable in that various combinations of dish-chutney probabilities lead to the same likelihood of the data. Thus, only the prior specification differentiates these entities (e.g., encouraging fewer concepts each with more words or more concepts each with fewer words.) Hierarchical clustering models often propose identifiability constraints such as the fact that components that correspond to the same class should be closer to each other than to components corresponding to other classes. In our model of Sec. 3, we avoid such explicit constraints and instead incorporate global information which reduces the sensitivity of the model to nBP hyperparameter settings. We do so in a way that maintains exchangeability (and thus computational tractability) of the model.

### 3 Capturing Superwords with Nested Beta Processes

Given a document collection of  $D$  documents and a vocabulary of  $V$  words, we wish to model the concepts therein as superwords. In particular, this entails identifying which concepts  $j$  are present in each document  $d$  (indicated by  $\hat{c}_j^{(d)} = 1$ ), along with which words  $i$  are active in concept  $j$  for document  $d$  (indicated by  $\hat{f}_{ji}^{(d)} = 1$ ). These binary variables are the nested beta process features described in Sec. 2, with  $\hat{\mathbf{c}}$  representing the chosen dishes and  $\hat{\mathbf{f}}$  the chosen chutneys. In particular, we assume a prior  $B \sim BP(1, BP(1, Q))$ ,

**Given:**  $Q = \sum_i \lambda_i \delta_{q_i}$ , where  $\lambda_i \in [0, 1]$  and  $q_i$  come from our vocabulary.

$(\omega, \gamma) \sim nBP(\alpha_\omega, Q)$

**for all documents**  $d = 1, \dots, D$  **do**

**for all concepts**  $j = 1, \dots, V$  **do**

$\pi_j^{(d)} \sim \text{Gamma}(\alpha_\pi, 1)$

$\hat{c}_j^{(d)} \sim \text{Bernoulli}(\omega_j)$

**for all**  $i = 1, \dots, V$  **do**

$\hat{f}_{ji}^{(d)} \sim \text{Bernoulli}(\gamma_{ji})$

**for all words**  $n$  **do**

$z_n^{(d)} \sim \pi^{(d)} \odot \hat{\mathbf{c}}^{(d)} / \sum_{j: \hat{c}_j^{(d)} = 1} \pi_j^{(d)}$

$w_n^{(d)} | z_n^{(d)} = z \sim \theta \odot \hat{\mathbf{f}}_z^{(d)} / \sum_{i: \hat{f}_{zi}^{(d)} = 1} \theta_i$

**for all semantic features**  $k$  **do**

$\sigma_k^2 \sim \text{InvGamma}(\alpha_\sigma, \beta_\sigma)$

$X_{kj} \sim N(0, \sigma_k^2 / K)$ , where  $K$  is a positive scalar

$Y_k^T \sim N((X_k \Phi)^T, \sigma_k^2 I_V)$

with a discrete base measure  $Q = \sum_{i=1}^V \lambda_i \delta_{q_i}$ , where  $\lambda_i \in [0, 1]$  and  $q_i$  come from our vocabulary.

Together,  $\hat{\mathbf{c}}$  and  $\hat{\mathbf{f}}$  define the makeup of the concepts and their presence in each document. However, to tie these superwords to the actual document text, we must specify a generative process for the observed words. First, we model the relative importance of each concept  $j$  in a document  $d$  as i.i.d. gamma-distributed random variables,  $\pi_j^{(d)} \sim \text{Gamma}(\alpha_\pi, 1)$ . We then associate the  $n$ th word in each document with a concept assignment  $z_n^{(d)}$ , drawn from a multinomial distribution proportional to  $\pi^{(d)} \odot \hat{\mathbf{c}}^{(d)}$  (where  $\odot$  refers to the element-wise Hadamard product).<sup>3</sup> As such,  $z_n^{(d)}$  can only take values  $j$  where  $\hat{c}_j^{(d)} = 1$ . Likewise, given an assignment  $z_n^{(d)} = j$ , a document generates its  $n$ th word,  $w_n^{(d)}$  from a multinomial distribution proportional to  $\theta \odot \hat{\mathbf{f}}_j^{(d)}$ . Here,  $\theta_i$  is a parameter of our model indicating the relative importance of words.<sup>4</sup> Our featural based model decouples concept presence in a document from its prevalence. Additionally, concepts that select overlapping sets of words need not have the same marginal probability of the shared word(s). A visual depiction of the graphical model as well as a summary of the full generative process is provided in Figure 2.

Our model as presented thus far suffers from the weak identifiability issues described in Sec. 2. In particular, the representational flexibility of the model can lead to many concepts, each containing few words, or a few concepts, each with many words, with little difference in the likelihood of the data between the two cases. We address this problem by incorporating semantic

<sup>3</sup>The non-zero elements of the resulting normalized distribution are Dirichlet distributed, with dimensionality determined by the number of ones in  $\hat{\mathbf{c}}$ .

<sup>4</sup>Alternatively,  $\theta_i$  can be modeled similarly to  $\pi_j^{(d)}$ , as gamma-distributed i.i.d. random variables.

information about our vocabulary in the manner described below. (cf. the supplemental material for an illustration of this problem on synthetic data.)

**Incorporating semantic knowledge.** We assume each word  $i$  in our vocabulary is associated with an *observed*, real-valued feature vector in an  $F$ -dimensional semantic space. Our fundamental assumption is that words appearing together in a concept should have similar semantic representations.

Beyond addressing identifiability concerns, explicitly modeling semantic features of our vocabulary allows us to elegantly incorporate a variety of side information to help guide the makeup of concepts. For instance, while the simplicity of the bag-of-words document representation provides many benefits in terms of computational efficiency, much structure is thrown away that could be useful to particular retrieval tasks. As such, if we want concepts to consist of words with related or synonymous meanings, we might consider to use sentence co-occurrence counts (e.g., in how many sentences do words  $w_1$  and  $w_2$  appear together?) as the basis for our semantic features. There is a long history of work in linguistics studying such distributional similarity, popularized by R. F. Firth, who stated that “you shall know a word by the company it keeps,” [7]. Rather than ignore this structural information, we can incorporate it as part of our semantic feature set, as we do in the experiments we consider in Sec. 5.

We model this idea by assuming that *concepts* are associated with *latent* semantic features in the same  $F$ -dimensional space. Words that often co-occur in concept  $j$  are expected to have semantic representations that are well-explained by the features for concept  $j$ . More formally, we consider an observed random matrix  $Y$  of dimensions  $F \times V$ , where each column  $Y_{\bullet i}$  corresponds to the semantic feature vector for word  $i$ . Likewise,  $X$  is a random matrix with  $F$  rows and a countably infinite number of columns, one per concept. As words can simultaneously be active in multiple concepts (e.g., “jaguar” can refer to both a car and an animal), we assume that the expected value of the feature representation for word  $i$ ,  $E[Y_{\bullet i}]$ , is equal to a weighted average over  $X_{\bullet j}$  for all concepts  $j$ , with weights proportional to the number of documents  $d$  with  $\hat{f}_{ji}^{(d)} = 1$ . In matrix notation, we write,  $E[Y] = X\Phi$ , where the weight matrix  $\Phi$  is deterministically computed from all  $\hat{\mathbf{c}}$  and  $\hat{\mathbf{f}}$ , such that  $\Phi_{ji} = \sum_{d:\hat{c}_j^{(d)}=1} \hat{f}_{ji}^{(d)} / \sum_j \sum_{d:\hat{c}_j^{(d)}=1} \hat{f}_{ji}^{(d)}$ .<sup>5</sup>

<sup>5</sup>It is important to note that in order to incorporate the semantic information into our generative model, we rely on the fact that our concepts are explicitly represented as *sparse* sets of words. The inclusion or exclusion of a word in a concept directly informs which concepts are respon-

By assuming independence of features and Gaussian noise, we can then specify the generative distribution for  $Y$  as described in Figure 2. In particular, we place conjugate normal and inverse gamma priors on the latent concept features  $X$  and the variance terms  $\sigma_k^2$ , respectively, allowing us to analytically marginalize them out for inference.

**Example: Learning multilingual concepts from images.** There are no modeling restrictions on the semantic data other than we expect the features to be real-valued and with zero mean. Hence, we can take advantage of this flexibility to model semantics in a variety of different forms, not limited to simply textual features. To demonstrate this flexibility, we consider the following toy problem: given a small collection of dessert recipes in English and German, downloaded from food blogs, can we learn concepts that are coherent *across* the two languages? Based on image-based semantic features, we address this problem without relying on parallel corpora or an explicit dictionary as in the multilingual topic modeling of [4, 13]. Specifically, assuming we have an image associated with each word in our vocabulary (e.g., from a Google images search), our semantic feature model encourages all concepts  $j$  that choose to include word  $i$  to have latent feature vectors that are similar to the image-based feature vector associated with word  $i$ . We hypothesize that, despite a lack of co-occurrence between the two languages within this small corpus, we should still obtain reasonable multilingual concepts, because an apple looks like an *apfel*, eggs look like *eier*, and so on.

Specifically, for each of 125 English and German vocabulary words, we first collect the top three search results from Google Images.<sup>6</sup> We transform these images following the approach of Oliva and Torralba [14], to get simple, 10-dimensional GIST-based features for each word, which we use as the semantic features. We then run our sampling procedure from Sec. 4 for 10,000 samples, which we use to infer the marginal probabilities of any two words being active in the same concept.

In order to quantitatively verify our hypothesis, we take this marginal probability matrix and, for each word  $w_i$ , rank all other words by probability of co-occurrence with  $w_i$ . We then create a ground truth set by finding all pairs of words in our vocabulary that are considered English-German translations of each other according to Google Translate. We come up with 18

sible for the semantic meaning of that word. To incorporate the same information in traditional topic modeling approaches, we would have all topics contributing to the semantic meaning of every word.

<sup>6</sup>Images for English words were retrieved from Google.com, and German words from Google.de, to avoid any internal translation that Google might otherwise do.

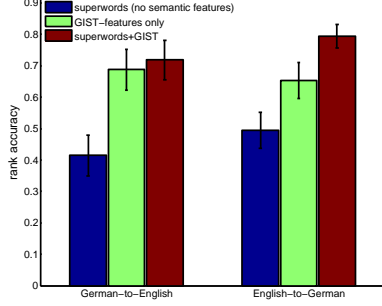


Figure 3: Rank accuracy of multilingual concepts inferred from English and German recipes; our model, combining image and corpus data, outperforms predictions based on either text or images alone.

such synonym pairs. We expand each set to include direct synonyms that differ only in case or number, giving us pairs of sets like  $\{egg, eggs\} \leftrightarrow \{eier\}$ . For each such pair, if  $w_g$  is a German translation of  $w_e$ , we see how high  $w_e$  is ranked on  $w_g$ 's marginal probability ranking, and compute the *rank accuracy*, which is the percentage of the words  $w_e$  is ranked higher than. We do this in both directions (from German to English and vice versa) for all 18 set-pairs. For comparison, we also compute rank accuracy when, rather than using our model, we rank words directly based on their  $L_2$ -distance in the 10-dimensional GIST feature space. Figure 3 shows that our model, combining corpus-based concept modeling with simple image-based semantic features improves performance over using the image features alone. Of course, as we expect, if we use our model without incorporating the images, we achieve poor performance, as there is little cross-language information in the text.

Anecdotally, we find several concepts that do in fact represent the types of cross-language coherence that we hoped for. For example, one concept consists primarily of kitchen tools and utensils,  $\{\text{whisk, rubber spatula, baking sheet, butter, tortenheber (cake server), schneebeesen (whisk), weizenmehl (wheat flour), mehl (flour)}\}$ , while another is heavy on mostly white and dry ingredients,  $\{\text{cornstarch, sugar, sugars, zucker (sugar), zuckers (sugars), vanillezucker (vanilla sugar), salz (salt), ricottakuchen (ricotta cake), ricotta, butter, pürierstab, teig (dough), mandel-zitronen-tarte, ofen, springform}\}$ . While neither of these concepts is perfect, they are still indicative of the flexibility and power of our semantic representation in guiding concept formation.

## 4 MCMC Computations

In order to perform inference in our model, we employ a Markov chain Monte Carlo (MCMC) method that

interleaves Metropolis-Hastings (MH) and Gibbs sampling updates. Given that we are primarily interested in the concept definitions ( $\hat{\mathbf{f}}$ ) and their existence and prevalence in each document ( $\hat{\mathbf{c}}$  and  $\boldsymbol{\pi}$ ), we marginalize out all other random variables, and end up with a collapsed sampler. This marginalization is analytically possible due to the conjugacy that exists throughout our model. Such collapsing is particularly critical to the performance of our sampler given the number of binary indicator variables in our model. For instance, if we sample  $\hat{c}_j^{(d)}$  without having marginalized out  $\mathbf{z}$ , we are forced to keep it set to 1 as long as any word  $n$  in document  $d$  has  $z_n^{(d)} = j$ .

Another important consideration is that, while our model represents an infinite number of concepts, only a finite number are ever instantiated at any given time. Thus, due to context-specific independence in our graphical model, we find that the variables  $\hat{\mathbf{f}}_j^{(d)}$  and  $\pi_j^{(d)}$  can be pruned from the model for cases where  $\hat{c}_j^{(d)} = 0$ , and sampled only on an as-needed basis.

At a high level, our sampling approach is as follows:

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### Algorithm 1 High-level MCMC inference procedure

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- 1: Sample  $\hat{\mathbf{c}}^{(d)} | \mathbf{w}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}^{(d)}, \hat{\mathbf{c}}^{(-d)}, Y$  for every document  $d$  using an MCMC procedure that proposes births/deaths for unique concepts.
  - 2: Sample  $\hat{f}_{ji}^{(d)} | \mathbf{w}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{c}}^{(d)}, \hat{\mathbf{f}}_{-(ji)}^{(d)}, Y$  for every document  $d$ , and every concept  $j$  present in  $d$ , and every word  $i$  in the vocabulary.
  - 3: Impute  $z_1^{(d)}, \dots, z_{N_d}^{(d)} | \mathbf{w}^{(d)}, \hat{\mathbf{c}}^{(d)}, \hat{\mathbf{f}}^{(d)}, \boldsymbol{\pi}^{(d)}$ .
  - 4: Use the imputed  $\mathbf{z}$  variables to aid in sampling  $\boldsymbol{\pi}^{(d)} | \mathbf{z}^{(d)}, \hat{\mathbf{c}}^{(d)}, \boldsymbol{\pi}_{-j}^{(d)}$ .
- 

For notational convenience, we write out two likelihood terms for sampling  $\hat{\mathbf{c}}^{(d)}$  and  $\hat{\mathbf{f}}^{(d)}$ . First,

$$P(\mathbf{w}^{(d)} | \hat{\mathbf{c}}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}^{(d)}) = \left( \sum_{k: \hat{c}_k^{(d)}=1} \pi_k^{(d)} \right)^{-N_d} \prod_{w \in \text{doc } d} \left( \sum_{z: \hat{c}_z^{(d)}=1 \wedge \hat{f}_{zw}^{(d)}=1} \frac{\theta_w \pi_z^{(d)}}{\sum_{l: \hat{f}_{zl}^{(d)}=1} \theta_l} \right)^{\#_w^{(d)}}, \quad (4)$$

where  $\#_w^{(d)}$  counts the occurrences of word  $w$  in document  $d$ , and  $N_d$  is the number of words in document  $d$ . Second, by marginalizing  $X$  and  $\sigma_k$  and recalling that  $\Phi$  is determined from  $\hat{\mathbf{c}}$  and  $\hat{\mathbf{f}}$ ,

$$P(Y | \Phi) \propto K^{|\mathcal{C}|F/2} |\Phi_{\mathcal{C}} \Phi_{\mathcal{C}}^T + KI_{|\mathcal{C}|}|^{-F/2} \prod_{k=1}^F \hat{\beta}_k^{-(\alpha_\sigma + V/2)}, \quad (5)$$

where  $\mathcal{C}$  is the set of active concepts, i.e.,  $\mathcal{C} = \{j :$





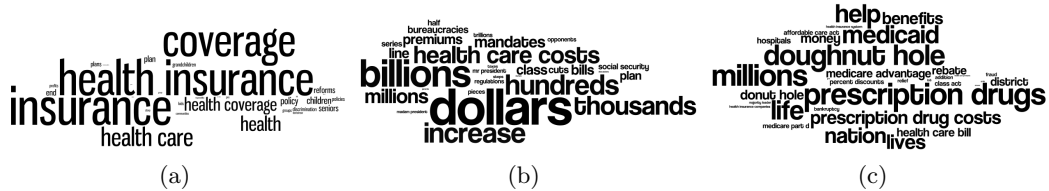


Figure 5: Word clouds representing three concepts generated from our model, applied to the Congressional data set, using sentence co-occurrence-based semantic features.

as we find both spellings) in Medicare’s prescription drug benefit, known as Medicare Part D.

In order to measure this coherency in a more quantitative manner, we conducted a user study where we presented participants with pairs of word clouds corresponding to ten key ideas from the health care debate (one cloud from our model and the other from HDP), and asked them to choose the cloud from each pair that provides the more coherent description of the word. For fairness, we asked a public policy expert to select ten words from our vocabulary representing the most salient ideas in the debate, and used these words to generate the word clouds.

For a given word, we found the HDP topic that assigned it the highest probability, as well as the concept from our model that included it the most times across the corpus. The word clouds were generated by removing the word in question and then displaying the remaining words proportional to their weight. To hide the identity of our approach from the participants, we truncate each HDP topic to have the same number of words as the corresponding concept from our model. This gives the HDP topics an illusion of sparsity that they do not naturally have, and hides one of the key advantages we have over topic models. For example, the “Obamacare” word clouds shown in the top row of Figure 1 are transformed to the ones in the bottom row for the purpose of this study. Despite handicapping our model in this manner, users found that our model produces more coherent concept representations than the HDP. Specifically, 73% of the 34 participants prefer our concepts to the HDP topics, with a mean preference of 5.74/10. Moreover, we note that the task itself relied on a subtle understanding of American politics. For example, overall, more participants preferred the HDP word cloud for “Obamacare” than ours, but when only considering participants who claimed to have followed the health care debate closely (and presumably understand that “Obamacare” is a term used pejoratively by Republicans), this preference is flipped to ours. More details on the study can be found in the supplemental material.

Finally, since our model allows concepts to have different words active in different documents, we can illustrate the flexibility of our prior by creating different

word clouds for the same concept, each representing a subset of documents. On this data set, a natural separation of the documents is to have two partitions, one representing Democrats and the other Republicans. Figure 4 shows two word clouds from our model representing the same concept, but one comes from Democrats and the other from Republicans. We can see the qualitative difference in the word weights between the two populations, with “discrimination” and “women” being more active in this concept for Democrats while “employer” and “market” being more associated with Republicans. We can find several similar anecdotes at the level of individual documents. For example, if we look at the leadership of the two parties, we find that in one concept, the word “Obamacare” is active for Republican Eric Cantor whereas “health care bill” is used for Democrat Chris Van Hollen.

## 6 Discussion

Motivated by critical problems in information retrieval, in this paper, we introduce a novel modeling technique for representing concepts in document collections. Popular generative models of text have tended to focused on representing the ideas in a document collection as probability distributions over the entire vocabulary, often leading to diffuse, uninformative topic descriptions of documents. In contrast, our discrete, sparse representation provides a focused, concise description that can, for example, enable IR systems to more accurately describe a user’s preferences. A second key contribution in this work is that our model naturally incorporates semantic information that is not captured by alternative approaches; for example, we demonstrated that images can be used to create multilingual concepts, without any other translation information. Such side information can often be easily obtained, and help guide topic models towards semantic meaningful predictions.

Despite the promise, our model suffers from common limitations faced by many Bayesian nonparametric methods. While incorporating semantic features was helpful from a modeling perspective, the matrix operations required to incorporate this data can be expensive. In experiments, we utilized parallelization and approximation techniques to reduce the running time,



but scalability remains a key aspect of future work.

We believe that the notion of superwords, characterized by sparse concepts and the incorporation of side information through semantic features, can significantly improve the effectiveness of IR techniques at capturing the nuances of users' preferences.

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## A Supplementary Material

In this appendix, we provide detailed derivations of our MCMC sampling procedure, as well as more details on the data we use for our experiments.

### A.1 MCMC Derivations

In our sampling procedure, we assume that  $\omega$ ,  $\gamma$  and  $\mathbf{z}$  are marginalized out of our model. However, in order to sample  $\hat{\mathbf{f}}$  and  $\boldsymbol{\pi}$ , we impute values for  $\mathbf{z}$ , which we discard at every iteration. As described in the main body of the paper, we employ a Gibbs sampler that features birth/death Metropolis Hastings steps for sampling  $\hat{\mathbf{c}}$  and a Gibbs-within-Gibbs sampler for jointly sampling  $\mathbf{X}$  and  $\hat{\mathbf{f}}$ .

#### A.1.1 Sample $\hat{\mathbf{c}}^{(d)} | \mathbf{w}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}, \hat{\mathbf{c}}^{(-d)}, \mathbf{Y}$

We assume that, at this moment, we have  $J$  total concepts active across all documents.  $J^{(-d)}$  are active in all other documents, while  $J_+^{(d)}$  are active only in this document (i.e.,  $J = J^{(-d)} + J_+^{(d)}$ ). Without loss of generality, we assume that the concepts are numbered such that the shared concepts  $(1, 2, \dots, J^{(-d)})$  come before the unique concepts  $(J^{(-d)} + 1, \dots, J)$ .

**Sampling a shared concept** Here, we consider sampling  $\hat{c}_j^{(d)} | \mathbf{w}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}, \hat{\mathbf{c}}_{-j}^{(d)}, \hat{\mathbf{c}}^{(-d)}, \mathbf{u}$  where  $j \in \{1, \dots, J^{(-d)}\}$ , i.e., concept  $j$  is shared. By Bayes' rule, as well as conditional independencies in the model, we have that:

$$\begin{aligned}
 & P(\hat{c}_j^{(d)} | \mathbf{w}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}, \hat{\mathbf{c}}_{-j}^{(d)}, \hat{\mathbf{c}}^{(-d)}, \mathbf{Y}) \\
 & \propto P(\mathbf{w}^{(d)}, \mathbf{Y} | \hat{\mathbf{c}}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}) \cdot P(\hat{c}_j^{(d)} | \hat{\mathbf{c}}_j^{(-d)}) \\
 & = P(\mathbf{w}^{(d)} | \hat{\mathbf{c}}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}^{(d)}) \cdot P(\mathbf{Y} | \Phi) \cdot P(\hat{c}_j^{(d)} | \hat{\mathbf{c}}_j^{(-d)}) \quad (9)
 \end{aligned}$$

By IBP exchangeability, we assume that the current document is the last one, allowing us to write the prior probability on  $\hat{c}_j^{(d)}$  (the last factor in this expression) as  $P(\hat{c}_j^{(d)} | \hat{\mathbf{c}}_j^{(-d)}) = m_j^{(-d)} / D$ , where  $m_j$  is the number of documents with  $\hat{c}_j^{(d)} = 1$ .

The first factor, which we call the text likelihood term, can be simplified as follows:

$$P(\mathbf{w}^{(d)} | \hat{\mathbf{c}}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}^{(d)}) \quad (11)$$

$$= \sum_{\mathbf{z}^{(d)}} \prod_{n=1}^{N_d} P(w_n^{(d)} | z_n^{(d)}, \hat{\mathbf{f}}^{(d)}) \cdot P(z_n^{(d)} | \hat{\mathbf{c}}^{(d)}, \boldsymbol{\pi}^{(d)}) \quad (12)$$

$$= \prod_{w \in \text{doc } d} \left( \sum_z \left( \frac{\theta_w \hat{f}_{zw}^{(d)}}{\sum_l \theta_l \hat{f}_{zl}^{(d)}} \right) \left( \frac{\pi_z^{(d)} \hat{c}_z^{(d)}}{\sum_k \pi_k^{(d)} \hat{c}_k^{(d)}} \right) \right)^{\#_w^{(d)}} \quad (13)$$

$$\propto \left( \sum_{k: \hat{c}_k^{(d)}=1} \pi_k^{(d)} \right)^{-N_d} \prod_{w \in \text{doc } d} \left( \sum_{z: \hat{c}_z^{(d)} \cdot \hat{f}_{zw}^{(d)}=1} \frac{\pi_z^{(d)}}{\sum_{l: \hat{f}_{zl}^{(d)}=1} \theta_l} \right)^{\#_w^{(d)}} \quad (14)$$

where  $\#_w^{(d)}$  is the count of word  $w$  in document  $d$ , and  $N_d$  is the total word count for document  $d$ . Note that if there is a word  $i$  that appears in document  $d$  such that the only concept that explains it is  $j$  (i.e.,  $\hat{f}_{ji}^{(d)} = 1$  but  $\hat{f}_{ki}^{(d)} = 0$  for all  $k \neq j$ ), then  $\hat{c}_j^{(d)}$  must be set to 1.

The second factor,  $P(\mathbf{Y} | \Phi)$ , which brings in influence from the semantic features, is derived at the end of this supplementary material.

We sample from this conditional distribution using a Metropolis Hastings step, where we have a deterministic proposal that flips the current value of  $\hat{c}_j^{(d)}$  from  $c$  to  $\bar{c}$ . Specifically, we flip  $\hat{c}_j^{(d)}$  with the following acceptance probability:

$$\rho(\bar{c} | c) = \min \left\{ \frac{P(\hat{c}_j^{(d)} = \bar{c} | \mathbf{w}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}, \hat{\mathbf{c}}_{-j}^{(d)}, \hat{\mathbf{c}}^{(-d)}, \mathbf{u})}{P(\hat{c}_j^{(d)} = c | \mathbf{w}^{(d)}, \boldsymbol{\pi}^{(d)}, \hat{\mathbf{f}}, \hat{\mathbf{c}}_{-j}^{(d)}, \hat{\mathbf{c}}^{(-d)}, \mathbf{u})}, 1 \right\}. \quad (15)$$

Note that if we consider flipping  $\hat{c}_j^{(d)}$  from 0 to 1, we will need to first sample values for  $\hat{f}_j^{(d)}$  and  $\pi_j^{(d)}$  from their priors, since they wouldn't otherwise exist. We sample  $\pi_j^{(d)}$  from  $\text{Gamma}(\alpha_\pi, 1)$  and sample  $\hat{f}_j^{(d)}$  from its prior in Equation 47.

**Sampling unique concepts** Let  $\hat{\mathbf{c}}_+^{(d)}$  be the current unique concepts for document  $d$ , and  $\hat{\mathbf{f}}_+^{(d)}$  and  $\boldsymbol{\pi}_+^{(d)}$  be their associated parameters. (Let  $\hat{\mathbf{c}}_-^{(d)}$ ,  $\hat{\mathbf{f}}_-^{(d)}$  and  $\boldsymbol{\pi}_-^{(d)}$  be the same for shared concepts.)

To sample these unique concepts, we'll use a birth/death proposal distribution, which factors as follows:

$$q(\hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{f}}_+^{(d)'}, \boldsymbol{\pi}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)}, \hat{\mathbf{f}}_+^{(d)}, \boldsymbol{\pi}_+^{(d)}} = q_c(\hat{\mathbf{c}}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)}} \quad (16)$$

$$\cdot q_f(\hat{\mathbf{f}}_+^{(d)' | \hat{\mathbf{f}}_+^{(d)}, \hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)}} q_\pi(\boldsymbol{\pi}_+^{(d)' | \boldsymbol{\pi}_+^{(d)}, \hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)}).$$

In our IBP, the  $D$ th customer is supposed to sample  $\text{Poisson}(\alpha_\omega/D)$  new dishes, and therefore the probability of a concept birth (which we call  $\eta(J_+^{(d)})$ ) is the probability that such a draw would result in more than the current number of unique concepts,  $J_+^{(d)}$ . In particular, we define  $\eta(J_+^{(d)}) = 1 - \text{PoissonCDF}(J_+^{(d)}; \alpha_\omega/D)$ , for  $J_+^{(d)} > 0$ . If

there are no unique concepts, then we are forced to propose a birth, and as such,  $\eta(0) = 1$ .

Thus, the proposal  $q_c(\hat{\mathbf{c}}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)})$  adds a new concept with probability  $\eta(J_+^{(d)})$ , and kills off each of the current  $J_+^{(d)}$  concepts with probability  $\frac{1-\eta(J_+^{(d)})}{J_+^{(d)}}$ . The proposals for  $\mathbf{f}$  and  $\boldsymbol{\pi}$  draw from their priors ( $\text{Bernoulli}(\lambda)$  and  $\text{Gamma}(\alpha_\pi, 1)$ , respectively) for a concept birth, and otherwise deterministically maintain the current values for the existing concepts.

Given this definition for our birth/death proposal, we can now write down our Metropolis Hastings step for sampling the unique  $\hat{\mathbf{c}}_+^{(d)}$  and associated parameters. In particular, we accept the Metropolis Hastings acceptance ratio is given by

$$r(\hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{f}}_+^{(d)'}, \boldsymbol{\pi}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)}, \hat{\mathbf{f}}_+^{(d)}, \boldsymbol{\pi}_+^{(d)}) = \frac{P(\hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{f}}_+^{(d)'}, \boldsymbol{\pi}_+^{(d)' | \mathbf{w}^{(d)}, \hat{\mathbf{c}}_-^{(d)}, \hat{\mathbf{c}}_-^{(-d)}, \hat{\mathbf{f}}_-^{(d)}, \boldsymbol{\pi}_-^{(d)}, \hat{\mathbf{f}}_-^{(-d)}, \mathbf{Y})}{P(\hat{\mathbf{c}}_+^{(d)}, \hat{\mathbf{f}}_+^{(d)}, \boldsymbol{\pi}_+^{(d)} | \mathbf{w}^{(d)}, \hat{\mathbf{c}}_-^{(d)}, \hat{\mathbf{c}}_-^{(-d)}, \hat{\mathbf{f}}_-^{(d)}, \boldsymbol{\pi}_-^{(d)}, \hat{\mathbf{f}}_-^{(-d)}, \mathbf{Y})} \cdot \frac{q(\hat{\mathbf{c}}_+^{(d)}, \hat{\mathbf{f}}_+^{(d)}, \boldsymbol{\pi}_+^{(d)} | \hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{f}}_+^{(d)'}, \boldsymbol{\pi}_+^{(d)'})}{q(\hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{f}}_+^{(d)'}, \boldsymbol{\pi}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)}, \hat{\mathbf{f}}_+^{(d)}, \boldsymbol{\pi}_+^{(d)}}. \quad (17)$$

Using Bayes' Rule and conditional independencies of our model, we can rewrite the first fraction as follows:

$$\frac{P(\mathbf{w}^{(d)} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)'}, [\boldsymbol{\pi}_-^{(d)} \boldsymbol{\pi}_+^{(d)'}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)'})]}{P(\mathbf{w}^{(d)} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)}, [\boldsymbol{\pi}_-^{(d)} \boldsymbol{\pi}_+^{(d)}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)}])} \cdot \frac{P(\mathbf{Y} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)'}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)'})]}{P(\mathbf{Y} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)}])} \cdot \frac{P(\hat{\mathbf{c}}_+^{(d)'}) P(\hat{\mathbf{f}}_+^{(d)'}) P(\boldsymbol{\pi}_+^{(d)'})}{P(\hat{\mathbf{c}}_+^{(d)}) P(\hat{\mathbf{f}}_+^{(d)}) P(\boldsymbol{\pi}_+^{(d)})} \quad (18)$$

Likewise, by definition, we can factor the second fraction as follows:

$$\frac{q_c(\hat{\mathbf{c}}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)'}) q_f(\hat{\mathbf{f}}_+^{(d)' | \hat{\mathbf{f}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)}) q_\pi(\boldsymbol{\pi}_+^{(d)' | \boldsymbol{\pi}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)})}{q_c(\hat{\mathbf{c}}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)'}) q_f(\hat{\mathbf{f}}_+^{(d)' | \hat{\mathbf{f}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)}) q_\pi(\boldsymbol{\pi}_+^{(d)' | \boldsymbol{\pi}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{c}}_+^{(d)})} \quad (19)$$

Plugging in the corresponding terms, if we propose a birth, we can simplify the acceptance ratio to:

$$r(\hat{\mathbf{c}}_+^{(d)'}, \hat{\mathbf{f}}_+^{(d)'}, \boldsymbol{\pi}_+^{(d)' | \hat{\mathbf{c}}_+^{(d)}, \hat{\mathbf{f}}_+^{(d)}, \boldsymbol{\pi}_+^{(d)}) = \frac{P(\mathbf{w}^{(d)} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)'}, [\boldsymbol{\pi}_-^{(d)} \boldsymbol{\pi}_+^{(d)'}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)'})]}{P(\mathbf{w}^{(d)} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)}, [\boldsymbol{\pi}_-^{(d)} \boldsymbol{\pi}_+^{(d)}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)}])} \cdot \frac{\text{Poisson}(J_+^{(d)} + 1; \alpha_\omega/D)(1 - \eta(J_+^{(d)} + 1))}{\text{Poisson}(J_+^{(d)}; \alpha_\omega/D)(J_+^{(d)} + 1)\eta(J_+^{(d)})} \cdot \frac{P(\mathbf{Y} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)'}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)'})]}{P(\mathbf{Y} | [\hat{\mathbf{c}}_-^{(d)} \hat{\mathbf{c}}_+^{(d)}, [\hat{\mathbf{f}}_-^{(d)} \hat{\mathbf{f}}_+^{(d)}])} \quad (20)$$

Likewise, if we propose to kill a concept, we have exactly the same form for the acceptance ratio, replacing the third line by:

$$\frac{\text{Poisson}(J_+^{(d)} - 1; \alpha_\omega/D) J_+^{(d)} \eta(J_+^{(d)} - 1)}{\text{Poisson}(J_+^{(d)}; \alpha_\omega/D) (1 - \eta(J_+^{(d)}))}. \quad (21)$$

### A.1.2 Impute $z_1^{(d)}, \dots, z_{N_d}^{(d)} | \mathbf{w}^{(d)}, \hat{\mathbf{c}}^{(d)}, \hat{\mathbf{f}}^{(d)}, \boldsymbol{\pi}^{(d)}$

For  $n = 1, 2, \dots, N_d$ :

By Bayes' rule:

$$P(z_n^{(d)} = z | w_n^{(d)} = w, \hat{\mathbf{c}}^{(d)}, \hat{\mathbf{f}}^{(d)}, \boldsymbol{\pi}^{(d)}) \propto P(w_n^{(d)} = w | z_n^{(d)} = z, \hat{\mathbf{f}}^{(d)}) \cdot P(z_n^{(d)} = z | \boldsymbol{\pi}^{(d)}, \hat{\mathbf{c}}^{(d)}). \quad (22)$$

Note that if  $\hat{c}_z^{(d)} = 0$  or  $\hat{f}_{zw}^{(d)} = 0$ , the probability of assigning  $z_n^{(d)} = z$  is 0. Therefore, when imputing the value of  $z_n^{(d)}$  (associated with word  $w$ ), we only need to consider values  $z$  such that  $\hat{c}_z^{(d)} = 1$  and  $\hat{f}_{zw}^{(d)} = 1$ . In this case, we sample  $z$  from:

$$\left( \frac{\theta_w}{\sum_{i: \hat{f}_{zi}^{(d)}=1} \theta_i} \right) \left( \frac{\pi_z^{(d)}}{\sum_{j: \hat{c}_j^{(d)}=1} \pi_j^{(d)}} \right). \quad (23)$$

Note that the numerator of the first factor and the denominator of the second factor are the same across all values of  $z$ , so we sample  $z_n^{(d)} = z$  (for  $z$  such that  $\hat{c}_z^{(d)} = 1$  and  $\hat{f}_{zw}^{(d)} = 1$ ) proportional to:

$$\frac{\pi_z^{(d)}}{\sum_{i: \hat{f}_{zi}^{(d)}=1} \theta_i}. \quad (24)$$

### A.1.3 Sample $\pi_j^{(d)} | \mathbf{z}^{(d)}, \boldsymbol{\pi}_{-j}^{(d)}, \hat{\mathbf{c}}^{(d)}$

One can think of the concept frequency random variables  $\pi_j^{(d)}$  as utility random variables aimed at modeling the concept frequency *distribution*

$$\tilde{\pi}^{(d)} = \frac{\boldsymbol{\pi}^{(d)} \odot \hat{\mathbf{c}}^{(d)}}{\sum_j \pi_j^{(d)} \hat{c}_j^{(d)}}. \quad (25)$$

Specifically, only considering the non-zero components of  $\tilde{\pi}^{(d)}$  (specified by the  $\hat{c}_j^{(d)} = 1$ ), this distribution is Dirichlet distributed and the gamma random variables are used in constructing a draw from this distribution. The purpose of utilizing the  $\pi_j^{(d)}$  in place of working with  $\tilde{\pi}^{(d)}$  directly is because the dimensionality of the underlying Dirichlet distribution is changing as  $\hat{c}^{(d)}$  changes and thus we can maintain an infinite collection of gamma random variables that are simply accessed during the sampling procedure.

We have multinomial observations  $\mathbf{z}^{(d)}$  (representing the word-concept assignments) from the concept frequency distribution  $\tilde{\pi}^{(d)}$ . Due to the inherent conjugacy of multinomial observations to a Dirichlet prior, the posterior of  $\tilde{\pi}^{(d)}$  is (using a slight abuse of notation):

$$\tilde{\pi}^{(d)} | \mathbf{z}^{(d)}, \hat{\mathbf{c}}^{(d)} \sim \text{Dir}([\alpha_\pi + n_1^{(d)}, \alpha_\pi + n_2^{(d)}, \dots] \odot \hat{\mathbf{c}}^{(d)}). \quad (26)$$

Once again, we can work in terms of our utility random variables  $\pi_j^{(d)}$  to form a draw from the desired Dirichlet posterior (again, along the non-zero components). Specifically, we draw

$$\pi_j^{(d)} | \mathbf{z}^{(d)} \sim \text{Gamma}(\pi_j^{(d)}; \alpha_\pi + n_j^{(d)}, 1). \quad (27)$$

If  $\hat{c}_j^{(d)} = 0$ , necessarily we will not have any counts of concept  $j$  in document  $d$  (i.e.,  $n_j^{(d)} = 0$ ), implying that the distribution of these utility random variables remains the same. Thus, in practice we only need to resample the utility random variables  $\pi_j^{(d)}$  for which  $\hat{c}_j^{(d)} = 1$ .

### A.1.4 Sample $\hat{f}_{ji}^{(d)} | \mathbf{w}^{(d)}, \hat{\mathbf{f}}_{-(ji)}^{(d)}, \hat{\mathbf{f}}^{(-d)}, \mathbf{z}^{(d)}, \hat{\mathbf{c}}$

First, we note that  $\hat{\mathbf{f}}_j^{(d)}$  only exists in documents  $d$  where  $\hat{c}_j^{(d)} = 1$ . (If  $\hat{c}_j^{(d)} = 0$ , it means that all observations  $\mathbf{w}^{(d)}$  are independent of  $\hat{\mathbf{f}}_j^{(d)}$ , and thus such  $f$  nodes can be pruned.) Second, while, due to conjugacy, we can write the conditional distribution for  $\hat{f}_{ji}^{(d)}$  without using  $\mathbf{z}^{(d)}$  (as was the case when sampling  $\hat{\mathbf{c}}$ ), we use the imputed  $\mathbf{z}^{(d)}$  here for computational efficiency.

We have the following:

$$P(\hat{f}_{ji}^{(d)} | \mathbf{w}^{(d)}, \hat{\mathbf{f}}_{-(ji)}^{(d)}, \hat{\mathbf{f}}^{(-d)}, \mathbf{z}^{(d)}, \hat{\mathbf{c}}) \quad (28)$$

$$= \int_0^1 P(\hat{f}_{ji}^{(d)}, \gamma_{ji} | \mathbf{w}^{(d)}, \hat{\mathbf{f}}_{-(ji)}^{(d)}, \hat{\mathbf{f}}^{(-d)}, \mathbf{z}^{(d)}, \hat{\mathbf{c}}) d\gamma_{ji} \quad (29)$$

$$= P(\mathbf{w}^{(d)} | \hat{\mathbf{f}}^{(d)}, \mathbf{z}^{(d)}) P(\mathbf{Y} | \hat{\mathbf{c}}, \hat{\mathbf{f}}) \int_0^1 P(\hat{f}_{ji}^{(d)}, \gamma_{ji} | \hat{\mathbf{f}}_{ji}^{(-d)}) d\gamma_{ji} \quad (30)$$

$$\propto \left( \prod_{n=1}^{N_d} P(w_n^{(d)} | \hat{\mathbf{f}}^{(d)}, z_n^{(d)}) \right) P(\mathbf{Y} | \hat{\mathbf{c}}, \hat{\mathbf{f}}) \int_0^1 P(\hat{\mathbf{f}}_{ji}^{(d: \hat{c}_j^{(d)}=1)}, \gamma_{ji}) d\gamma_{ji}. \quad (31)$$

The first factor in this expression is the corpus likelihood term, assuming  $z_n^{(d)}$ . We can simplify this as follows:

$$\prod_{n=1}^{N_d} P(w_n^{(d)} | \hat{\mathbf{f}}^{(d)}, z_n^{(d)}) \quad (32)$$

$$= \prod_{n=1}^{N_d} \frac{\theta_{w_n^{(d)}} \hat{f}_{z_n^{(d)}}^{(d)} w_n^{(d)}}{\sum_l \theta_l \hat{f}_{z_n^{(d)}l}^{(d)}} \quad (33)$$

$$\propto \prod_{n: z_n^{(d)}=j} \frac{\theta_{w_n^{(d)}} \hat{f}_{jw_n^{(d)}}^{(d)}}{\sum_l \theta_l \hat{f}_{jl}^{(d)}} \quad (34)$$

$$= \left( \sum_{l: \hat{f}_{jl}^{(d)}=1} \theta_l \right)^{-n_j^{(d)}} \prod_{n: z_n^{(d)}=j} \theta_{w_n^{(d)}} \hat{f}_{jw_n^{(d)}}^{(d)}. \quad (35)$$

Recall that the second factor is the semantic likelihood term, and is derived at the end of this supplementary material.

Finally, the integral, representing the prior probability on  $\hat{f}_{ji}$ , is simplified as follows:

$$\int_0^1 P(\hat{\mathbf{f}}_{ji}^{(d:\hat{c}_j^{(d)}=1)}, \gamma_{ji}) d\gamma_{ji} \quad (36)$$

$$= \int_0^1 P(\hat{\mathbf{f}}_{ji}^{(d:\hat{c}_j^{(d)}=1)} | \gamma_{ji}) P(\gamma_{ji}) d\gamma_{ji} \quad (37)$$

$$= \int_0^1 \prod_{d:\hat{c}_j^{(d)}=1} P(\hat{f}_{ji}^{(d)} | \gamma_{ji}) P(\gamma_{ji}) d\gamma_{ji} \quad (38)$$

$$= \int_0^1 \gamma_{ji}^{m_{ji}} (1 - \gamma_{ji})^{m_j - m_{ji}} \text{Beta}(\gamma_{ji}; \lambda_i, 1 - \lambda_i) d\gamma_{ji} \quad (39)$$

$$= \frac{1}{B(\lambda_i, 1 - \lambda_i)} \int_0^1 \gamma_{ji}^{m_{ji} + \lambda_i - 1} (1 - \gamma_{ji})^{m_j - m_{ji} - \lambda_i} d\gamma_{ji} \quad (40)$$

$$= \frac{B(m_{ji} + \lambda_i, m_j - m_{ji} + 1 - \lambda_i)}{B(\lambda_i, 1 - \lambda_i)} \quad (41)$$

$$\propto \Gamma(m_{ji} + \lambda_i) \Gamma(m_j - m_{ji} + 1 - \lambda_i), \quad (42)$$

where  $m_{ji}$  is the number of documents with both  $\hat{c}_j^{(d)} = 1$  and  $\hat{f}_{ji}^{(d)} = 1$ . Thus, if  $\hat{f}_{ji}^{(d)} = 1$ , we have:

$$\Gamma(m_{ji}^{(-d)} + \lambda_i + 1) \Gamma(m_j - m_{ji}^{(-d)} - \lambda_i) = \quad (43)$$

$$= (m_{ji}^{(-d)} + \lambda_i) \Gamma(m_{ji}^{(-d)} + \lambda_i) \Gamma(m_j - m_{ji}^{(-d)} + \lambda_i), \quad (44)$$

and if  $\hat{f}_{ji}^{(d)} = 0$ , we have:

$$\Gamma(m_{ji}^{(-d)} + \lambda_i) \Gamma(m_j - m_{ji}^{(-d)} + 1 - \lambda_i) = \quad (45)$$

$$= \Gamma(m_{ji}^{(-d)} + \lambda_i) (m_j - m_{ji}^{(-d)} - \lambda_i) \Gamma(m_j - m_{ji}^{(-d)} - \lambda_i). \quad (46)$$

After some cancellation, we get the following:

$$P(\hat{f}_{ji}^{(d)} = 1 | \hat{\mathbf{f}}_{ji}^{(-d)}) = \frac{m_{ji}^{(-d)} + \lambda_i}{m_j}, \quad (47)$$

$$P(\hat{f}_{ji}^{(d)} = 0 | \hat{\mathbf{f}}_{ji}^{(-d)}) = \frac{m_j - m_{ji}^{(-d)} - \lambda_i}{m_j}. \quad (48)$$

To recap, we sample  $\hat{f}_{ji}^{(d)}$  as follows:

1. if  $n_j^{(d)} = 0$ , we sample  $\hat{f}_{ji}^{(d)}$  proportional to  $P(\mathbf{Y} | \hat{\mathbf{c}}, \hat{\mathbf{f}}) P(\hat{f}_{ji}^{(d)} | \hat{\mathbf{f}}_{ji}^{(-d)})$ .
2. if  $n_j^{(d)} > 0$  but  $n_{ji}^{(d)} = 0$ , we sample proportional to  $P(\mathbf{Y} | \hat{\mathbf{c}}, \hat{\mathbf{f}}) P(\hat{f}_{ji}^{(d)} | \hat{\mathbf{f}}_{ji}^{(-d)}) \left( \sum_{l:\hat{f}_{jl}^{(d)}=1} \theta_l \right)^{-n_j^{(d)}}$ .
3. else, if  $n_j^{(d)} > 0$  and  $n_{ji}^{(d)} > 0$ , set  $\hat{f}_{ji}^{(d)} = 1$  with probability 1.

### A.1.5 Determine $p(\mathbf{Y} | \Phi)$

Recall that  $F$  denotes the dimensionality of our semantic features. If the number of concepts were finite with  $J$  concepts, we could specify

$$X | \Sigma \sim MN(M, \Sigma, K) \quad (49)$$

$$\Sigma \sim \text{IW}(n_0, S_0), \quad (50)$$

where  $MN$  denotes a matrix normal distribution and  $\text{IW}$  and inverse Wishart. Here,  $M$  defines the mean matrix which  $\Sigma$  and  $K$  define the left and right covariances of dimensions  $F \times F$  and  $J \times J$ , respectively. Typically,  $K$  is assumed to be diagonal with  $K = \text{diag}(k_1, \dots, k_F)$ .

Using the fact that our features are independently Gaussian distributed, we can write

$$\mathbf{Y} | X, \Sigma, \Phi \sim MN(X\Phi, \Sigma, I_J) \quad (51)$$

The prior for  $X, \Sigma$  above is conjugate to this likelihood, so we can analytically compute the marginal likelihood. Standard matrix normal inverse Wishart conjugacy results yield

$$P(\mathbf{Y} | \Phi) = \frac{|K|^{F/2} |S_0|^{n_0/2} 2^{VF/2}}{(2\pi)^{V/2} |S_{\bar{y}\bar{y}}|^{F/2} |S_0 + S_{y|\bar{y}}|^{(n_0+V)/2}} \cdot \prod_{\ell=1}^F \frac{\Gamma(\frac{n_0+1-\ell}{2} + \frac{V}{2})}{\Gamma(\frac{n_0+1-\ell}{2} + \frac{V}{2})}. \quad (52)$$

Here,

$$S_{y|\bar{y}} = S_{yy} - S_{y\bar{y}} S_{\bar{y}\bar{y}}^{-1} S_{\bar{y}y} \quad (53)$$

$$S_{\bar{y}\bar{y}} = \Phi \Phi' + K \quad (54)$$

$$S_{y\bar{y}} = \mathbf{Y} \Phi' + M K \quad (55)$$

$$S_{yy} = \mathbf{Y} \mathbf{Y}' + M K M'. \quad (56)$$

In the case of an unbounded number of concepts, we restrict our attention to noise covariances  $\Sigma$  that are diagonal ( $\Sigma = \text{diag}(\sigma_1^2, \dots, \sigma_F^2)$ ). This implies

$$X_{kj} | \sigma_k^2, k_j \sim N(M_{kj}, \sigma_k^2/k_j) \quad (57)$$

$$Y_{ki} | X, \Phi, \sigma_k^2 \sim N(X_k \cdot \Phi_i, \sigma_k^2), \quad (58)$$

independently for all  $i, j, k$ .

Although we are not working with a finite model, the  $\Phi$  matrix implicitly truncates our model. Let  $X_C$  represent the set of latent concept features associated with the instantiated concepts and  $\Phi_C$  the non-zero columns of  $\Phi$  associated with the active concepts. Then, our model above is equivalent to

$$\mathbf{Y} | X_C, \Phi_C \sim MN(X_C \Phi_C, \Sigma, I_C). \quad (59)$$

Using the marginal likelihood formula of Eq. (52), and simplifying based on the factorization of the likelihood across the dimensions of our feature space, yields

$$P(\mathbf{Y} | \Phi) = \frac{|K_{CC}|^{F/2} \beta_{\sigma}^{\alpha_{\sigma} F} \Gamma(\alpha_{\sigma} + V/2)^F}{|\Phi_C \Phi_C^T + K_{CC}|^{F/2} (2\pi)^{FV/2} \Gamma(\alpha_{\sigma})^F} \prod_{k=1}^F \hat{\beta}_k^{-(\alpha_{\sigma} + V/2)}. \quad (60)$$

### A.2 Synthetic Example

The phenomenon of weak identifiability is illustrated in Figure 6. We generate synthetic data for 100 documents, assuming the underlying concept representation depicted in Figure 6(a). We then run our Gibbs sampler (cf. Sec. 4) for 1,000 samples in order to infer the concept definitions for each document. Figure 6(b) shows the average concept

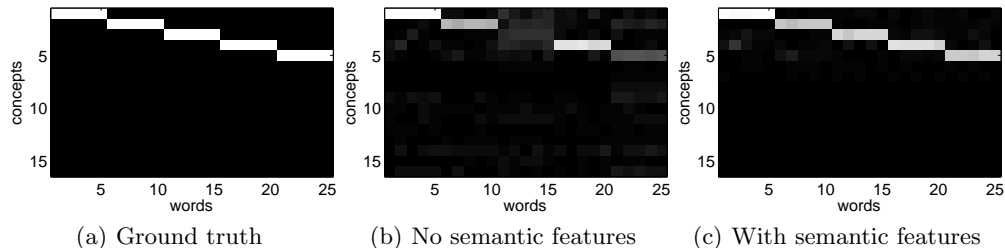


Figure 6: Synthetic example illustrating weak identifiability of the nested beta process and how we overcome it using semantic features. For each active concept, the average  $\hat{\mathbf{f}}^{(d)}$  vector over all documents  $d$  is plotted, with white indicating presence of a word in a concept, black indicating absence, and gray for average values of  $\hat{\mathbf{f}}^{(d)}$  in between 0 and 1.

definitions  $\hat{\mathbf{f}}^{(d)}$  across all documents, for the final sample. We see that while most of the concepts are correctly recovered, the sampler has difficulty reconstructing the concept representing words 11 to 15. However, by incorporating additional semantic information about our vocabulary in the manner described below, we are able to properly recover all five concepts, as seen in Figure 6(c).

### A.3 User study results

We filtered user study participants to make sure they had followed the health care debate at the least at the level of reading the headlines. Of these 34 participants, we first measured how many of the ten questions resulted in a favorable vote for our model as compared to HDP, and found it to be 5.74 on average:

```
Num: 34
Average: 5.7352941176471
Standard Dev: 1.5398400132598
T-test 95% conf. interval:
[5.2176965657521, 6.252891669542]
```

We then asked, how many participants preferred our words to HDP, treating each participant as a Bernoulli sample, and ignoring the ties:

```
Num wins: 16
Num losses: 6
Binomial mean: 0.72727272727273
Binomial std: 0.44536177141512
Binomial Sign Test 95% conf. interval:
[0.54116788781464, 0.91337756673082]}
```

### A.4 Sampler details

We initialize  $\hat{\mathbf{c}}$  and  $\hat{\mathbf{f}}$  in our sampler from a simple k-means clustering of the words using the semantic features. It is interesting to see how, after many samples, how a concept’s definition changes from the initialization. Figure 7 shows one particular concept, and how our model reweights the words and adds/removes words from the initial cluster.

We ran our sampler with the following hyperparameter settings:

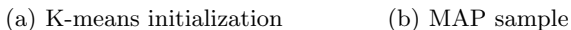


Figure 7: Concept changes from initialization



**In this user study, you will be presented with 10 pairs of word clouds describing various topics from the U.S. health care reform debate of 2009-2010, and will be asked a few questions. This study should take 5-10 minutes.**

## Do you follow US politics?

Yes No

**How much did you pay attention to the US health care debate of 2009-2010?**

- I followed it closely.
- I paid attention to the main headlines.
- Very little, if at all.

**1. Which of the two word clouds more coherently describes the concept COSTS?**

this one



this one

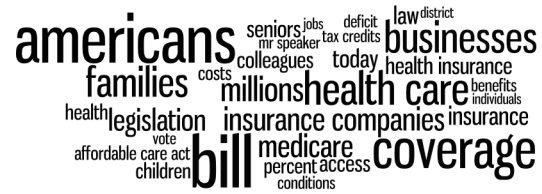


**2. Which of the two word clouds more coherently describes the concept REPEAL?**

this one



this one

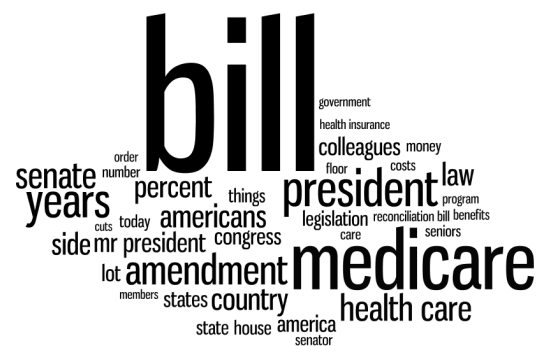


**3. Which of the two word clouds more coherently describes the concept PREMIUMS?**

this one



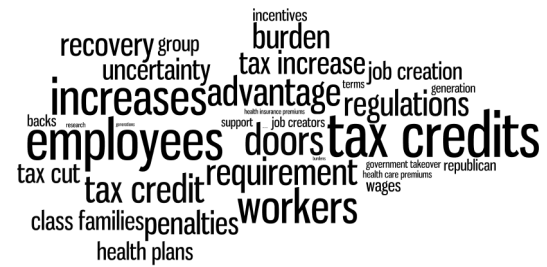
this one



**4. Which of the two word clouds more coherently describes the concept MANDATES?**

this one

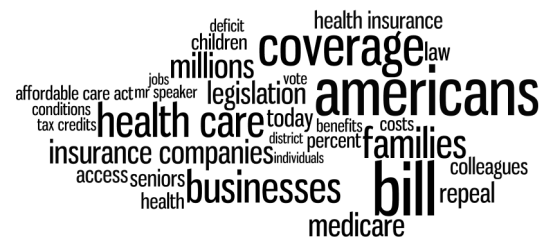
this one



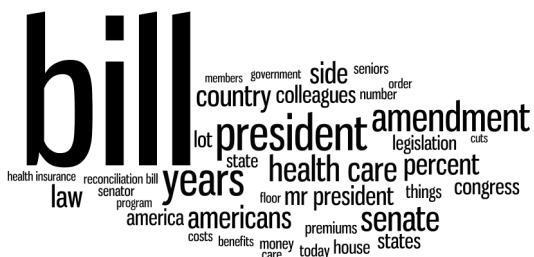
this one



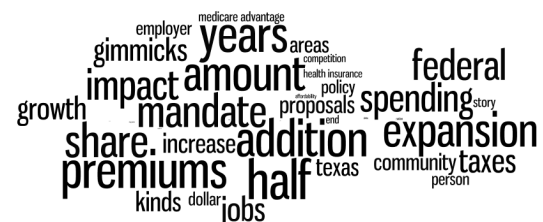
this one



this one



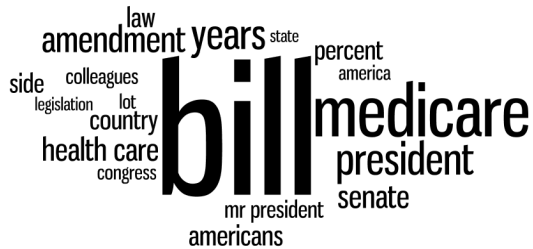
this one



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7. Which of the two word clouds more coherently describes the concept  
STATES?

[this one](#)



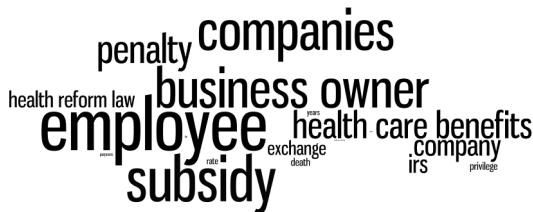
[this one](#)



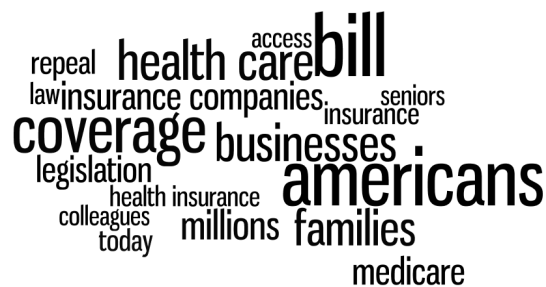
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8. Which of the two word clouds more coherently describes the concept  
EMPLOYERS?

[this one](#)



[this one](#)



---

9. Which of the two word clouds more coherently describes the concept  
OBAMACARE?

this one

care  
costs mr speaker  
doctors  
government plan  
patients

this one

money  
mistake  
bill challenge  
medicine

10. Which of the two word clouds more coherently describes the concept  
**COVERAGE?**

this one

medicare  
health care  
bill  
americans  
families  
businesses  
legislation  
tax credits  
district today  
insurance companies  
benefits law  
millions  
seniors  
health  
percent costs  
repeal  
mr speaker  
conditions  
vote  
affordable care act deficit  
children  
access  
colleagues  
individuals

this one

policies  
businesses  
insurance  
health reform  
health coverage  
bankruptcy  
assistance  
discrimination  
consumer protections  
business owners  
health care reform bill  
thousands  
district  
health insurance  
constituents  
repeal  
retirees  
market  
health care costs

**Make sure you've made a selection for each one!**

Submit